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Numerical simulation of bubbly two-phase flow in a narrow channel

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Abstract

An advanced numerical simulation method on fluid dynamics – lattice-Boltzmann (LB) method is employed to simulate the movement of Taylor bubbles in a narrow channel, and to investigate the flow regimes of two-phase flow in narrow channels under adiabatic conditions. The calculated average thickness of the fluid film between the Taylor bubble and the channel wall agree well with the classical analytical correlation developed by Bretherton. The numerical simulation of the behavior of the flow regime transition in a narrow channel shows that the body force has significant effect on the movement of bubbles with different sizes. Smaller body force always leads to the later coalescence of the bubbles, and decreases the flow regime transition time. The calculations show that the surface tension of the fluid has little effect on the flow regime transition behavior within the assumed range of the surface tension. The bubbly flow with different bubble sizes will gradually change into the slug flow regime. However, the bubbly flow regime with the same bubble size may be maintained if no perturbations on the bubble movement occur. The slug flow regime will not change if no phase change occurs at the two-phase interface.

Keywords: Narrow channel; Taylor bubble; Lattice-Boltzmann; Flow regime transition; Two-phase flow

1. Introduction

Microscale two-phase flow and heat transfer are of great interest in many industrial applications, such as, thermo-siphons for cooling of electronic components, micro-heat-pipe systems, materials processing, oil industry and biotechnology.

Numerous theoretical and experimental investigations on the thermodynamics of two-phase flow in narrow channels have been performed. Some progress has been obtained concerning the flow pattern of two-phase flow in narrow channels, and some aspects of thermal hydraulics of boiling in narrow channels, such as pressure drop and average heat transfer coefficient. Many correlations have been obtained to model these physical phenomena. It is found that the pressure drop and average heat transfer coefficient are significantly dependent on the flow pattern, which is similar to what has been found for the boiling in large channels. However, the behavior of flow regime transition of two-phase flow in narrow channels is much different from that in large channels. So far, little information on flow regime transitions of two-phase flow in narrow channels can be found in the literature.

The lattice-Boltzmann (LB) method has been developed quite recently [5]. The LB method recovers the Navier–Stokes equations in the incompressible flow limit. Since the LB method can be considered as a mesoscopic approach, lying in between microscopic molecular dynamics and conventional macroscopic fluid dynamics, it can be useful when microscopic statistics and macroscopic description of flow are important, e.g. in problems involving surface tension, capillarity and phase transition in multiphase multicomponent systems.

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Nomenclature

$\mathbf{e}, c$	lattice speed, m/s
Ca	capillary number
Eo	Eoẗvoš number
f, n	distribution function
F	force, N
Fr	Froude number
G	interaction strength
g	gravity acceleration, m/s^2
H	half hydraulic diameter, m
m	molecular weight of component
M	Morton number
n	particle distribution
P	pressure, N/m^2
t	time, s
u	velocity, m/s
x	coordinates, (x, y), m
y	film thickness, m

Greek symbols

ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3
τ	relaxation time
ψ	effective number density
σ	surface tension, N/m

Superscript and subscripts

a	index of discrete lattice velocity
b	the number of the discrete velocities
c	critical value
eq	equilibrium
g	gravity
S	the number of phases
σ	index of phase, surface tension
t	fluid–solid interaction

The LB code, named FlowLab, was developed at the Royal Institute of Technology [4]. The code performance was examined for a spectrum of fluid flow problems, including single and two-phase flow in porous media [8]. The code was also employed to simulate the flow regime transition characteristics under pool boiling conditions [9].

In this work, the FlowLab code is applied to simulate the movement of single Taylor bubbles and for regime transition of two-phase flow in a narrow channel.

2. Formulation of lattice-Boltzmann method

The idea of the LB approach originates from the kinetic theory of gases. The integro-differential Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{u} \cdot \nabla f = \sum(\mathbf{x}, \mathbf{u}, t) \tag{1}$$

describes the relation of the molecular distribution function $f(\mathbf{x}, \mathbf{u}, t)$, representing the number of molecules found in the phase space volume element $\Delta\Xi = \Delta x_i, \Delta u_i, \Delta E(u_i, x_i,\ i = 1, 3)$. The right-hand side of the Boltzmann equation (1) is the collision integral, which describes the source/sink of particles (due to collision) in the infinitesimal volume $\Delta\Xi$. The simplest model for the collision integral, which is valid in the case of small deviations of the system from the equilibrium state, has the following form:

$$\sum(\mathbf{x}, \mathbf{u}, t) = -\frac{1}{\tau}(f - f^{\mathrm{eq}}), \tag{2}$$

where τ has the meaning of relaxation time (towards equilibrium).

The discrete-velocity-Boltzmann equation for multicomponent and multiphase flow is written in the following form [6]:

$$n_a^\sigma(\mathbf{x} + \mathbf{e}_a, t + 1) - n_a^\sigma(\mathbf{x}, t) = -\frac{1}{\tau_\sigma} \cdot \left[n_a^\sigma(\mathbf{x}, t) - n_a^{\sigma(\mathrm{eq})}(\mathbf{x}, t)\right], \tag{3}$$

where $n_a^{\sigma(\mathrm{eq})}(\mathbf{x}, t)$ is the equilibrium distribution at (x, t). Superscript σ denotes the fluid component, $\sigma = 1, \ldots, S$ and subscript a denotes the lattice velocity direction, $a = 1, \ldots, b$. We should note, that Eq. (3) is normalized by the lattice spacing, Δx, and the reference lattice speed, $c = \Delta x/\Delta t$.

The functional form for the equilibrium distribution of a rectangular D2Q9 model, shown in Fig. 1, is chosen as

$$n_0^{\sigma(\mathrm{eq})}(\mathbf{x}) = t_0 n^\sigma(\mathbf{x}) \cdot \left[1 - \frac{3}{2}\mathbf{u}^2\right], \tag{4}$$

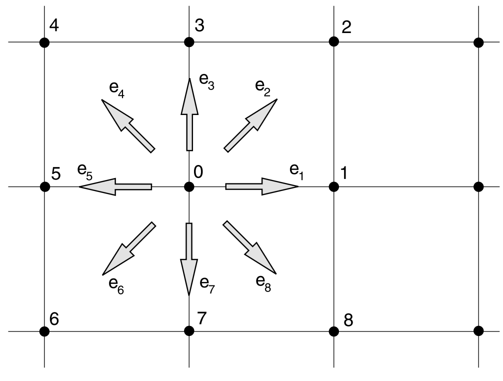

Fig. 1. Lattice geometry and velocity vectors of D2Q9 model.

$$n_a^{\sigma(\text{eq})}(\mathbf{x}) = t_i n^\sigma(\mathbf{x}) \cdot \left[1 + 3\mathbf{e}_a \cdot \mathbf{u} + \frac{3}{2}(3\mathbf{e}_a\mathbf{e}_a : \mathbf{uu} - \mathbf{u}^2)\right], \quad (5)$$

where $n^\sigma = \sum_{a=1}^{b} n_a^\sigma$, $t_0 = 4/9$ (the rest populations), $t_1 = 1/9$ (populations, moving in non-diagonal directions) and $t_2 = 1/36$ (diagonal directions). In the above expressions, arbitrary constant d_0 was chosen as 1/3 [3].

Physical quantities of flow, such as fluid density $\rho^\sigma(\mathbf{x}, t)$ and fluid velocity $\mathbf{u}^\sigma$, can be obtained from:

$$\rho^\sigma(\mathbf{x}, t) = \sum_a m^\sigma n_a^\sigma(\mathbf{x}, t) \quad (6)$$

$$\rho^\sigma(\mathbf{x}, t)\mathbf{u}^\sigma(\mathbf{x}, t) = \sum_\sigma m^\sigma \sum_a n_a^\sigma(\mathbf{x}, t)\mathbf{e}_a + \mathbf{F}_{\text{total}}^\sigma, \quad (7)$$

where m^σ is the molecular mass of the σth component, $\mathbf{F}_{\text{total}}^\sigma$ is the momentum contributed by the total force acting on the σth component:

$$\mathbf{F}_{\text{total}}^\sigma = \mathbf{F}_{\text{g}}^\sigma + \mathbf{F}_\sigma^\sigma + \mathbf{F}_{\text{t}}^\sigma, \quad (8)$$

where $\mathbf{F}_{\text{g}}^\sigma$, $\mathbf{F}_\sigma^\sigma$ and $\mathbf{F}_{\text{t}}^\sigma$ are momenta contributed by gravity, interaction between phases, and interaction of fluid with solid, respectively.

In the present work, the interaction potential model of Shan and Chen [6], which simulates the hydrodynamic interaction between the two phases, is employed

$$\mathbf{F}_\sigma^\sigma = -\psi^\sigma \sum_{\sigma'}^{S} G_{\sigma\sigma'} \sum_{a=0}^{b} \psi^{\sigma'}(\mathbf{x} + \mathbf{e}_a)\mathbf{e}_a, \quad (9)$$

where ψ^σ is a function of $n(\mathbf{x})$ and plays the role of the effective number density for component σ. $G_{\sigma\sigma'}$ is the interaction potential.

The model, developed by Martys and Chen [3] to describe the interaction between a fluid and a wall, is implemented in the FlowLab code:

$$\mathbf{F}_{\text{t}}^\sigma = -n^\sigma(\mathbf{x}) \sum_a G_{\text{t}}^\sigma s(\mathbf{x} + \mathbf{e}_a)(\mathbf{e}_a), \quad (10)$$

where G_{t}^σ is the fluid–solid interaction potential parameter, $s = 0$ or 1 for fluid or solid, respectively.

In the LB method, only fluid density is introduced directly. The kinematic viscosity of the fluid can be obtained from $v = (2\tau - 1)/6$, where τ is the relaxation time in the LB equation. By choosing proper τ, the viscosity of the fluid can be obtained. The surface tension of the fluid in two-phase flow can be evaluated from the interaction potential G_σ [6]. Fig. 2 shows the simulation results of surface tension for different interaction potential parameters by the LB method. In this calculation, the lattice number is 51×51, the density is assumed to be similar in the two fluids. The dependence of surface tension on the interaction potential G_σ is nearly linear in this model.

In the previous work [10], the FlowLab code was employed to simulate the two-phase flow regime transition behavior under pool boiling condition. Different flow regime transition mechanisms were represented numerically. Note that the time and length scales in this paper are dimensionless, and are expressed by time step number and the lattice number. All other physical parameters are dimensioned based on these two parameters.

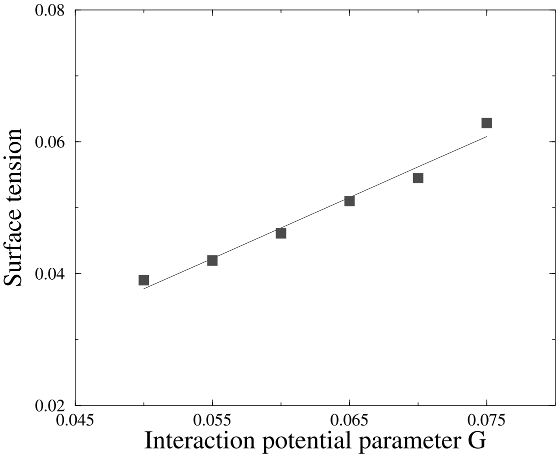

Fig. 2. Evaluation of surface tension for two-phase flow.

3. Numerical simulation on the bubble behavior in a narrow channel

The movement of a single Taylor bubble (slug) in a vertical narrow channel is simulated to study the effect of different physical parameters on the bubble movement.

A Taylor bubble in a narrow channel can be considered to be composed of two parts, a rounded nose region whose shape and dimensions are independent of the overall Taylor bubble length and a cylindrical section surrounded by an annular film of the continuous fluid.

White and Beardmore [7] developed a graphical correlation for the terminal velocity of slugs, which is quite accurate, and still employed to analyze the Taylor bubble movement. The graphical correlation employs three different dimensionless numbers: Eötvös number, *Eo*; Morton number, *M*; and Froude number, *Fr*:

$$Eo = \frac{g\Delta\rho d^2}{\sigma}, \quad M = \frac{g\mu^4\Delta\rho}{\rho^2\sigma^3}, \quad Fr = \frac{U}{\sqrt{gd}}\sqrt{\frac{\rho}{\Delta\rho}}, \quad (11)$$

where U is the terminal velocity of the bubble, ρ is the density of the bubble, σ is the surface tension, d is the channel diameter, and μ is the viscosity. According to the graphical correlation proposed by White and Beardmore [7], there are three different special cases:

(1) viscosity and surface tension forces are negligible ($M \leqslant 10^{-6}$ and $Eo > 100$), the Fr number is constant, which is about 0.35 for the bubble in vertical tube; (2) surface tension is dominant ($Eo < 3.4$), in this case, the slug remains motionless with its shape determined by a balance between hydrostatic and capillary forces; and (3) viscosity is dominant ($Eo > 70$ and $Fr < 0.05$), the terminal velocity for these conditions is given by $U = gd^2\Delta\rho/102\mu$. In the present paper, we are focusing on the case with small size of the channel, for which case the capillary force is dominant and the movement of the bubble is a function of many physical parameters. One of the most important parameters is the capillary number $Ca = U\mu/\sigma$.

Fig. 3 shows the typical movement behavior of a single Taylor bubble in a vertical narrow channel, calculated by the FlowLab code. The initial condition is static, without any force acting on the bubble. The gas phase is assumed to be a non-wetting fluid, while the liquid (continuous) phase is a wetting fluid, so no contact occurs between the gas phase and the channel wall surface. The mass flow rate of the liquid in the channel is assumed to be zero. It can be seen from Fig. 3 that the deformation of the bubble occurs only at the initial time period, after that the shape and movement of the bubble is stable, and a constant rising velocity is established.

Fig. 4 shows the interface shape of the single Taylor bubble in a vertical narrow channel for different capillary numbers. It can be seen that as the value of capillary number increases the stagnant flow region thickens and the wiggles vanish. The changes of the curvature of the interface of the meniscus of the bubble for different capillary numbers agree well with the numerical

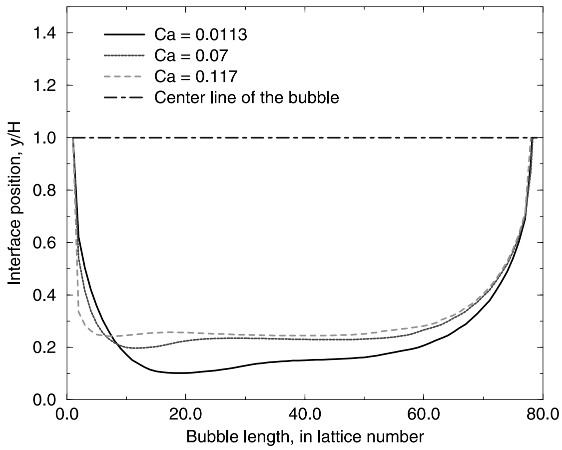

Fig. 4. The interface shapes of single Taylor bubble in a vertical narrow channel for different capillary numbers Ca.

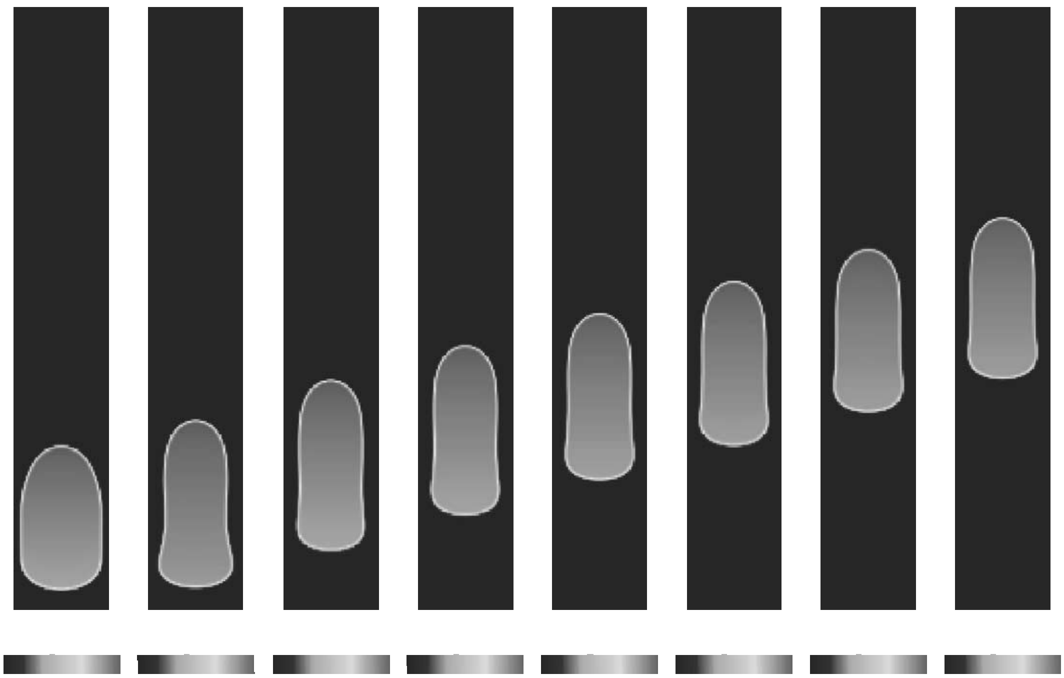

Fig. 3. The movement of a single Taylor bubble in a vertical narrow channel.

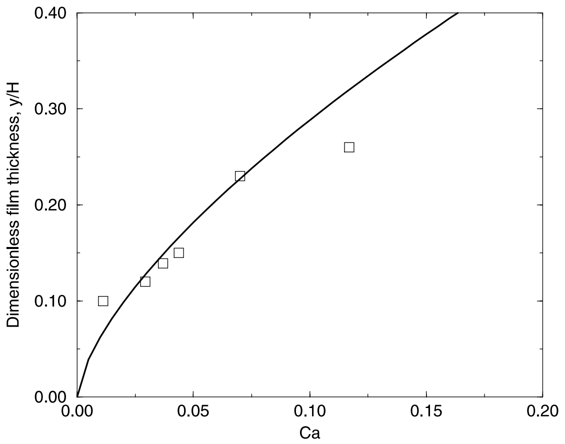

Fig. 5. The liquid film thickness between the bubble and the Taylor bubble. The line is the equation developed by Bretherton [1], the squares is the numerical results of the present calculation.

prediction by [2], which employed Galerkin/finite element discretization of the governing equations.

The thickness of the liquid film between the bubble and the liquid wall is also calculated. The results are compared with the analytical correlation developed by Bretherton [1]

$$\frac{y}{H} = 1.337Ca^{2/3}, \tag{12}$$

where y is the film thickness; H is one-half of the hydraulic diameter of the channel. It can be seen from Fig. 5 that a good agreement is achieved reasonably.

4. Numerical simulation of the flow regime transition

The flow regime of the two-phase flow in a narrow channel is investigated. Typically, there are three different regimes of two-phase flow in a narrow channel under boiling conditions as follows:

Isolated bubble flow. Bubbles detach from the nucleation sites and flow as discrete units in the liquid, as the flow proceeds further heat addition results in an increase in the number and size of the bubbles.

Confined bubble flow – Taylor bubble. Bubbles span the gap (in spaces confined in one dimension) or fill the channel (in spaces confined in two dimensions), they are separated from the wall by a layer of liquid which evaporates and causes the bubble to grow exponentially.

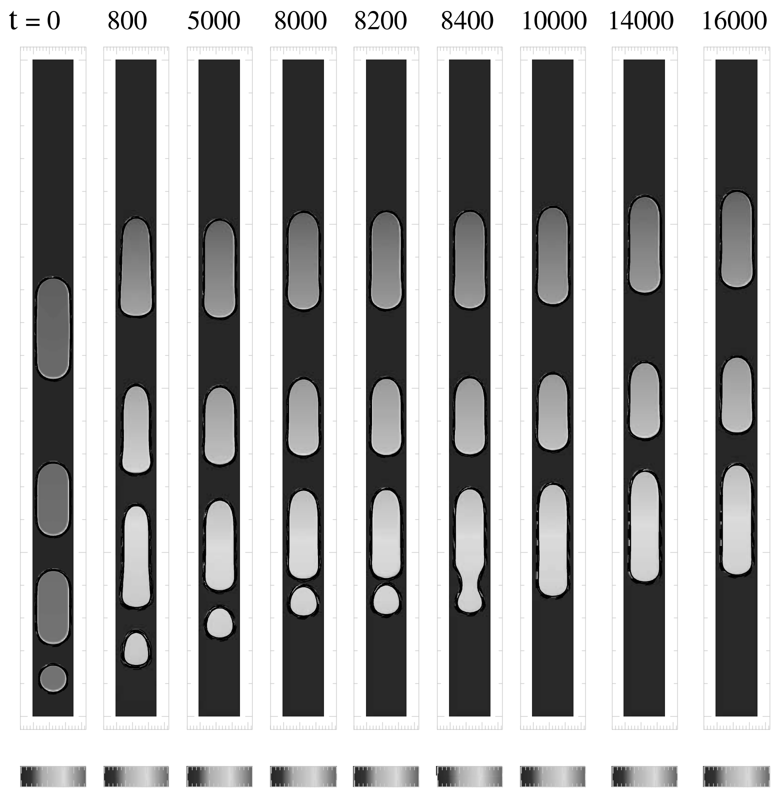

Fig. 6. The bubble movement in a narrow channel: surface tension 0.038, body force 0.0008.

The bubbles may be formed by isolated bubbles growing or coalescing, alternatively single bubbles may reach a sufficient size to be regarded as confined before becoming detached from their nucleation sites.

Annular-slug flow. As the confined bubbles expand, liquid in slug (Taylor bubble) between them is deposited on the channel wall and the flow becomes basically annular with random, irregular slugs of liquid interspersed with the vapor.

Generally, two different mechanisms of regime transition of two-phase flow in a narrow channel can be identified: one is the phase change, which causes the increase of the bubble size, hence changes the shape of the bubble and the flow regime; the another one is the coalescence of the bubbles. The latter is more important since the coalescence of bubbles changes the flow regime rapidly. In this paper, only the latter mechanism is investigated for the transition behavior of the two-phase flow regime in narrow channels. No heat transfer and phase change are considered between the two phases. All the calculations are two-dimensional. The effect of different physical parameters, e.g. body force, surface tension, bubble size, etc. on the flow regime transition behavior is investigated.

4.1. The effect of body force

Two different cases are calculated, however, with the same initial conditions and the same physical properties of the fluids, but with different body force acting on the bubble fluid. Consider four bubbles in the channel, three of them are big, and slug-like, only one of them is small. Figs. 6 and 7 show the calculated results with two different body forces: 0.0008 and 0.0012. It can be seen in both figures that only the small bubble merged with another slug which is moving ahead of it. Practically, the three slugs are moving at the same velocity in the channel. With no change of the volume of the bubbles, these three slugs will never coalesce. The regime will be slug flow. It can also be seen that the coalescence between the small bubble with the slug ahead is earlier in smaller body force condition than that in larger body force condition. This is because the velocity difference between the small bubble and the slug is larger in smaller body force condition than that in larger body force condition. It can be concluded that the transition from slug + bubble regime to pure slug regime will occur earlier in smaller body force condition than that in larger body force condition.

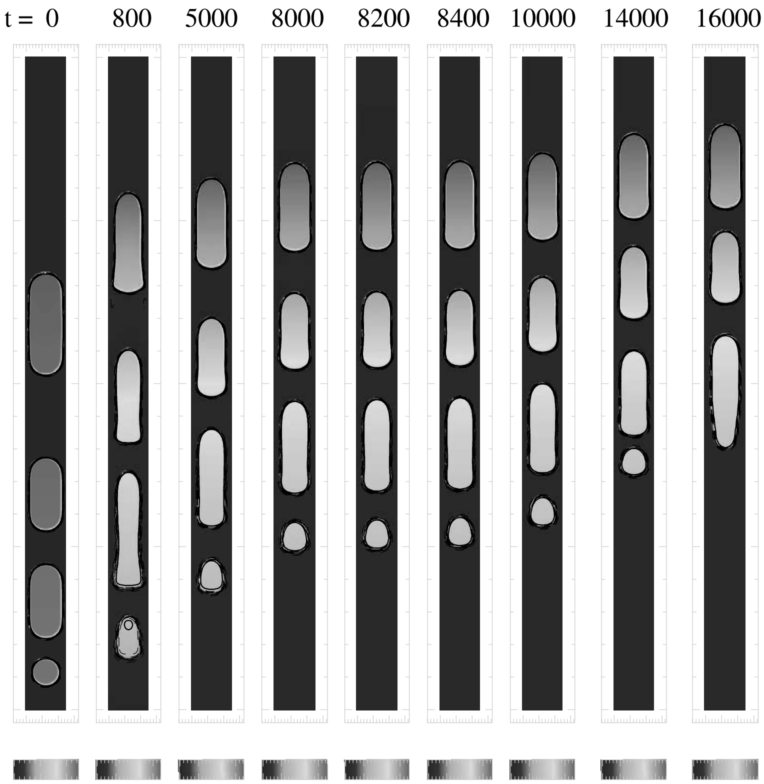

Fig. 7. The bubble movement in a narrow channel: surface tension 0.038, body force 0.0012.

4.2. The effect of surface tension

Surface tension is an important parameter for movement of Taylor bubbles, especially, in a narrow channel, in which the surface tension is a dominant parameter. Fig. 8 shows that calculation of the bubble movement in a narrow channel with the same condition as in Fig. 6 but a larger surface tension force. Within the present range of the change in the surface tension, the surface tension has little effect on the movement of the bubbles and the flow regime transition. The coalescence of the small bubble with the slug ahead of it, however, occurs a little later in Fig. 8. The difference is due to different acceleration in the initial period of the calculation. Different surface tension force at the two-phase interface leads to different deformation of the bubble at the beginning, and a somewhat different initial moving dynamics. After a certain time, the movement of the bubbles in these two cases does not differ greatly.

4.3. The effect of bubble size

It has been shown in the above-mentioned calculations that the movement of slugs with different size in narrow channels is almost identical, no flow regime transition will occur if no phase change is considered, In most cases, the bubbles generated, and detached from the channel wall, may not be small if the continuous fluid is moving. So the movement of the small bubbles, whose sizes are smaller than or of the same scale as the channel diameter, is of significant interest in terms of the flow regime transition.

Fig. 9 shows the calculated bubble movement in a narrow channel. The picture clearly indicates that the bubbles with different sizes having different velocities, the bubbles with smaller size is moving faster than those with larger size. After the coalescence of two bubbles, a larger bubble is formed. Before an elongated Taylor bubble is formed, the larger bubbles will try to coalesce with others. So finally the flow regime transitions to slug flow. It may be concluded from this calculation that if bubbles of the same size are moving in a channel, it is impossible for them to coalesce with each other and the bubbly flow regime will be maintained. However, any perturbations on the moving bubbles may change the flow regime. The large slugs will never merge with each other if no phase change occurs at the two-phase interface.

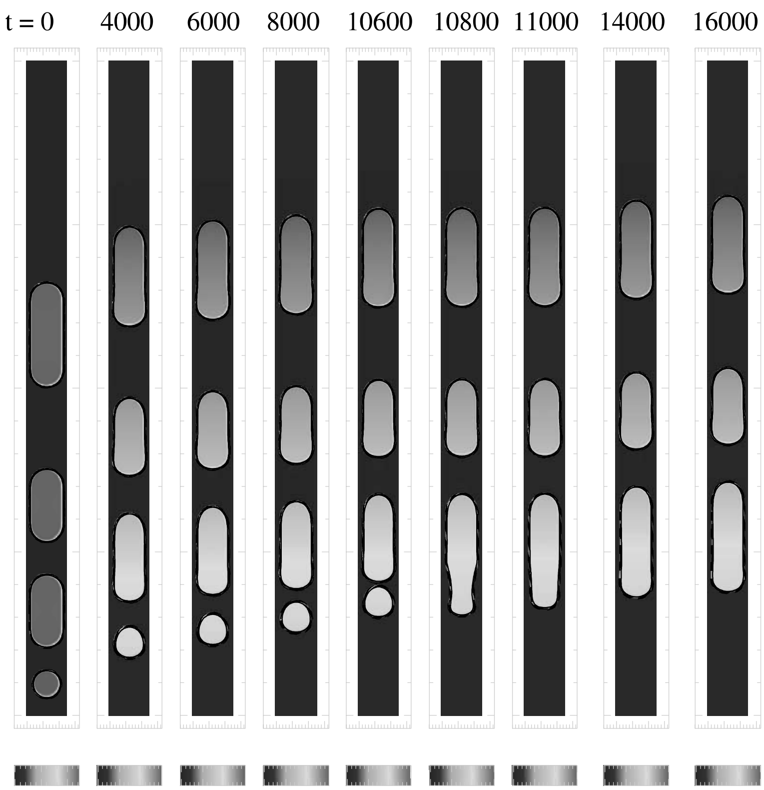

Fig. 8. The bubble movement in a narrow channel: surface tension 0.0475, body force 0.0008.

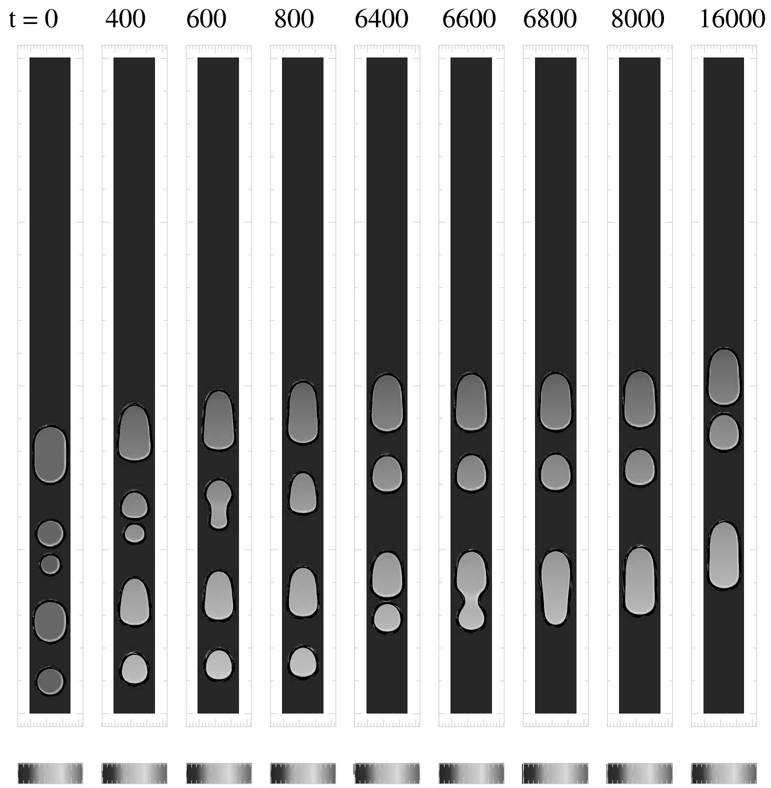

Fig. 9. The bubble movement in a narrow channel: surface tension 0.038, body force 0.0008.

5. Concluding remarks

In this paper, an advanced numerical simulation with LB method is employed to simulate the movement of Taylor bubbles and to investigate the regimes of the two-phase flow in a narrow channel.

Different aspects of the movement of a single Taylor bubble in a narrow channel are investigated by present calculations. The interface shape of single Taylor bubble meniscus agrees well with predictions by other numerical methods. The calculated results on the average film thickness of the liquid fluid between the Taylor bubble and the channel wall agree well with the classical analytical correlation developed by Bretherton.

The numerical simulation of the flow regime transition in the narrow channel shows that the body force has significant effect on the movement of bubbles with different sizes. Smaller body force always leads to the later coalescence of the bubble, and decreases the flow regime transition. The calculation shows that the surface tension of the fluid has little effect on the flow regime transition within the chosen range of the surface tension. Bubbly flow with different bubble sizes will gradually change into the slug flow regime. However, the bubbly flow regime with the identical bubble size may be maintained if no perturbations on the bubble moving behavior occur. The slug flow regime will not change if no phase change occurs at the two-phase interface.

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